**P425/1**

**PURE MATHEMATICS**

**Paper 1**

**June/July. 2022**

3 hours

**RESOURCEFUL MOCK EXAMINATIONS 2022**

**Uganda Advanced Certificate of Education**

PURE MATHEMATICS

**Paper 1**

3 hours

**INSTRUCTIONS TO CANDIDATES:**

*Answer **all** the **eight** questions in section A and any **five** from section **B**.*

*Any additional question(s) answered will **not** be marked.*

***All** necessary working **must** be shown clearly.*

*Begin each answer on a fresh page.*

*Silent non- programmable scientific calculators and mathematical tables with a list of formulae may be used.*

**TURN OVER**

## SECTION A: (40 MARKS)

*Attempt **all** the questions in this section.*

1. Expand $\sqrt{1+4x^2}$ in ascending powers of $x$ up to the term in $x^6$. Hence evaluate $\sqrt{1.04}$ correct to 4s.f. *(05 marks)*

2. The first and last terms of a geometric series are 2 and 2048. The sum of the series is 2730. Find the number of terms and the common ratio.

   *(05 marks)*

3. Given that $y = ln\left(\frac{1-3x}{1-2x}\right)$, show that $\frac{dy}{dx} = \frac{-1}{(3x-1)(2x-1)}$. *(05 marks)*

4. Given that $sinA = \frac{4}{5}$ and $cosB = \frac{5}{13}$ where $A$ is acute and $B$ is reflex, find without using mathematical tables or a calculator the value of $\tan(A-B)$. *(05 marks)*

5. Evaluate: $\int_0^{\pi/2} cos3xcos2xdx$ *(05 marks)*

6. A point P moves such that its distance from A is twice its distance from B. A and B are fixed points $(-2,1)$ and $(5,6)$ respectively. Find the locus of P and describe it fully. *(05 marks)*

7. A point C divided AB externally with $A(2,3,-1)$ and $B(-1,2,3)$ in the ratio $4:3$ . Find the equation of line through C and perpendicular to the plane $3x - 4y + 2z = 5$. *(05 marks)*

8. Find the area bounded by the curve $y = 2 + x - x^2$ and the line $y = 2$.

   *(05 marks)*

## SECTION B: (60 MARKS)

*Attempt **only five** questions in this section.*

9. (a) Given that $z_1 = 2 - 3i$ and $z_2 = 3 + 5i$, find $\frac{z_1}{z_2}$ in polar form. *(06 marks)*

(b) Given $Z = x + yi$,find the equation of locus of $Z$ if $|Z + 2 - 3i| = |Z - 3 + 4i|$. Represent Z on an Argand diagram. *(06 marks)*

10. (a) Prove that $log_c ab = log_c a + log_c b$. Hence solve the equation $log_3(x - 2) + log_3(x + 2) = 3$ *(06 marks)*

(b)The equation $3x^2 - 7x - 1 = 0$ has roots $\alpha$ and $\beta$. Find the values of $(\alpha - \beta)^2$ and $\alpha^4 + \beta^4$. Hence form an equation with integral coefficients whose roots are $(\alpha - \beta)^2$ and $\alpha^4 + \beta^4$. *(06 marks)*

11. (a) Evaluate: $\int_0^3 \frac{x^2}{\sqrt{x+1}} dx$. *(06 marks)*

(b) Find the value of $\int_0^1 x^2 e^{-2x} dx$. *(06 marks)*

12. The line $l_1$ has equation $r = 5i + 8j + k + t(i + 8k)$ , line $l_2$ has equation $r = i + j + \alpha(3i + 24k)$ and plane P has equation $2x - 2y - z = 5$.

(a) Show $l_1$ and $l_2$ are parallel to each other. *(03 marks)*

(b) Obtain the point of intersection of line $l_1$ and the plane $P$. *(05 marks)*

(c) Find the angle between the plane $P$ and line $l_2$. *(04 marks)*

13. (a) Given $y = \frac{(3x+1)^4}{(5x-2)^3}$, show that $\frac{dy}{dx} = \frac{3(3x+1)^3(5x-13)}{(5x-2)^4}$. *(06 marks)*

(b) A hollow cone of base radius 10cm and height 10cm is held with its vertex downwards. The cone is initially empty when water is poured into it at the rate of $4\pi cm^3 s^{-1}$. Find the rate of increase in depth of water in the cone 18 seconds after pouring has commenced. *(06 marks)*

14.(a) Show that $2cotA - sin2A = cot^2 Asin2A$. *(05 marks)*

(b) Express $3cos2\theta - 8sin\theta cos\theta$ in the form $Rcos(2\theta + \alpha)$ where $\alpha$ is a positive acute angle. State the maximum value of $y = 4 + 3cos2\theta - 8sin\theta cos\theta$ and the smallest positive value of $\theta$ when it occurs.

*(07 marks)*

15.(a) show that the equation $y^2 - 4y - 8x + 12 = 0$ represents a parabola. State its focus and directrix and hence sketch it. *(05 marks)*

(b)Find the equation of the normal to the parabola $y^2 = 4ax$ at the point $(at^2, 2at)$. This normal meets the $x - axis$ at A and $y - axis$ at B. Find the equation of locus of M the mid-point of AB. *(07 marks)*

16.(a) Find the general solution of the differential equation $x^2 y \frac{dy}{dx} - 2x = 1$.

*(04 marks)*

(b) A beaker containing water at $100^0 C$ is placed in a room which has a constant temperature of $20^0 C$. The rate of cooling is proportional to the excess temperature between temperature of the room and the liquid. If after 5 minutes, the temperature of the water is $60^0 C$, what will be it after 10 minutes? *(08 marks)*

## GOOD LUCK